

The Condensate for $SU(2)$ Yang-Mills Theory in 1+1 Dimensions Coupled to Massless Adjoint Fermions*

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Abstract

We consider $SU(2)$ Yang-Mills theory in 1+1 dimensions coupled to massless adjoint fermions. With all fields in the adjoint representation the gauge group is actually $SU(2)/Z_2$, which possesses nontrivial topology. In particular, there are two distinct topological sectors and the physical vacuum state has a structure analogous to a θ vacuum. We show how this feature is realized in light-front quantization, using discretization of x^- as an infrared regulator. We find exact expressions for the vacuum states and construct the analog of the θ vacuum. We calculate the bilinear condensate of the model. We argue that this condensate does not effect the spectrum of the massless theory but gives the string tension of the massive theory.

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I. INTRODUCTION

We shall focus our attention here on $SU(2)$ Yang-Mills theory coupled to adjoint fermions. This theory is known to have nontrivial vacuum structure; as first pointed out by Witten [1]. For gauge group $SU(2)$ the model has a Z_2 topological structure and possesses two distinct vacuum states which we calculate explicitly. The physical vacuum state is a state analogous to a θ vacuum that allows the cluster property to be satisfied. There is a nonvanishing bilinear condensate [2] which we calculate. We will argue that the condensate that we find is not related to the condensate found in the equal-time theory [3]. We are interested in this theory because it is an example of a light-cone quantized theory with a non-trivial vacuum. This theory is interesting for a variety of additional reasons. It has been shown to have the same massive spectrum as QCD_{1+1} with two colors of fundamental matter, where the topology is trivial and there is a unique vacuum state [4]. We will argue that the condensate does not affect the massive spectrum of the massless adjoint theory but is related to the string tension in the theory with massive adjoint fermions [5]

II. $SU(2)$ GAUGE THEORY COUPLED TO ADJOINT FERMIONS: BASICS

Let us now consider $SU(2)$ gauge theory coupled to adjoint fermions in one space and one time dimension. Since all fields transform according to the adjoint representation, gauge transformations that differ by an element of the center of the group actually represent the same transformation and so should be identified. Thus the gauge group of the theory is $SU(2)/Z_2$, which has nontrivial topology: $\Pi_1[SU(2)/Z_2] = Z_2$, so that we expect two topological sectors. This situation differs from the case when the matter fields are in the fundamental representation, where the gauge group is $SU(2)$ and the first homotopy group is trivial.

The Lagrangian for the theory is

$$\mathcal{L} = -\frac{1}{2}Tr(F^{\mu\nu}F_{\mu\nu}) + \frac{i}{2}Tr(\bar{\psi}\gamma^\mu \overleftrightarrow{D}_\mu \psi), \quad (2.1)$$

where $D_\mu = \partial_\mu + ig[A_\mu, \]$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$. A convenient representation of the gamma matrices is $\gamma^0 = \sigma^2$ and $\gamma^1 = i\sigma^1$, where σ^a are the Pauli matrices. With this choice the Fermi field may be taken to be hermitian.

The matrix representation of the fields makes use of the $SU(2)$ generators $\tau^a = \sigma^a/2$. It is convenient to introduce a color helicity, or Cartan, basis, defined by

$$\tau^\pm \equiv \frac{1}{\sqrt{2}}(\tau^1 \pm i\tau^2) \quad , \quad [\tau^+, \tau^-] = \tau^3 \quad , \quad [\tau^3, \tau^\pm] = \pm\tau^\pm. \quad (2.2)$$

Lower helicity indices are defined by $\tau_\pm = \tau^\mp$. In terms of this basis, matrix-valued fields are decomposed as, for example,

$$\begin{aligned} A^\mu &= A_3^\mu \tau^3 + A_+^\mu \tau^+ + A_-^\mu \tau^- \\ \Psi_{R/L} &= \psi_{R/L} \tau^3 + \phi_{R/L} \tau^+ + \phi_{R/L}^\dagger \tau^- \end{aligned} \quad (2.3)$$

where $A^{\mu,\pm} \equiv (A_1^\mu \pm iA_2^\mu)/\sqrt{2}$; $A^{\mu,\pm} = A_\mp^\mu$; $(A_+^\mu)^\dagger = A_-^\mu$ and $\phi_{R/L} \equiv (\Psi_{R/L}^1 - i\Psi_{R/L}^2)/\sqrt{2}$.

We shall regulate the theory by requiring that the gauge field A^μ be periodic and the right-handed Fermi field be antiperiodic in x^- . The left-handed fermion Ψ_L is taken to be antiperiodic in the coordinate x^+ . In most cases we will not present explicitly the left handed contributions. These are presented and discussed in more detail elsewhere [6].

The Fock space representation for the Fermi fields is obtained by Fourier expanding Ψ_R on $x^+ = 0$,

$$\psi_R = \frac{1}{2^{1/4}\sqrt{2L}} \sum_n \left(a_n e^{-ik_n^+ x^-} + a_n^\dagger e^{ik_n^+ x^-} \right) \quad (2.4)$$

$$\phi_R = \frac{1}{2^{1/4}\sqrt{2L}} \sum_n \left(b_n e^{-ik_n^+ x^-} + d_n^\dagger e^{ik_n^+ x^-} \right) . \quad (2.5)$$

and similarly Ψ_L on $x^- = 0$ with a_n, b_n, d_n and x^- replaced by $\alpha_n, \beta_n, \delta_n$ and x^+ . Here the sums run over the positive half-odd integers and $k_n^\pm = n\pi/L$. The Fourier modes obey the standard anti-commutation relations

$$\{a_n^\dagger, a_m\} = \{b_n^\dagger, b_m\} = \{d_n^\dagger, d_m\} = \delta_{n,m} \quad (2.6)$$

with all mixed anti-commutators vanishing. The fermionic Fock space is generated by acting with the various creation operators to a vacuum state $|0\rangle$.

III. CURRENT OPERATORS

The current operators for this theory are

$$J^+ \equiv J^R = -\frac{1}{\sqrt{2}} [\Psi_R, \Psi_R] . \quad (3.1)$$

and similarly for $J^- \equiv J^L$. To avoid confusion, we shall henceforth always write the currents with R or L in place of the upper Lorentz index and the color helicity index either up or down with $J_3 = J^3$ and $J_- = J^+$.

These expressions for the currents are ill-defined as they stand since they contain the product of operators at the same point. This is a common problem and occurs in the expression for the Poincaré generators as well. We shall regulate these expressions by point splitting while maintaining gauge invariance and then take the splitting to zero after removing the singularities. The singularities give rise to additional contributions, so called gauge corrections. We find that the current J^R acquires a gauge correction

$$J^R = \tilde{J}^R - \frac{g}{2\pi} V , \quad (3.2)$$

where \tilde{J}^R is the naive normal-ordered current. The left handed current has an anomaly $-\frac{g}{2\pi} A$.

It is convenient to Fourier expand these currents and discuss the the properties of their components. We write

$$\tilde{J}^{R,a} = \sum_{N=-\infty}^{\infty} C_N^a e^{-i\pi N x^- / L} \quad (3.3)$$

where a is a color index and the sums are over the integers. The coefficients in the expansion for the left handed currents are D_N^a . It is well known that these Fourier components satisfy a Kac-Moody algebra with level one [7]. We shall discuss this explicitly for the C_N^a ; an identical set of relations holds for the D_N^a , with appropriate substitutions.

In terms of the Fock operators, we find,

$$\begin{aligned} C_N^3 &= \sum_{n=\frac{1}{2}}^{\infty} b_n^\dagger b_{N+n} - \sum_{n=\frac{1}{2}}^{\infty} d_n^\dagger d_{N+n} - \sum_{n=\frac{1}{2}}^{N-\frac{1}{2}} b_n d_{N-n} \\ C_N^+ &= \sum_{n=\frac{1}{2}}^{\infty} a_n^\dagger d_{N+n} - \sum_{n=\frac{1}{2}}^{\infty} b_n^\dagger a_{N+n} - \sum_{n=\frac{1}{2}}^{N-\frac{1}{2}} d_n a_{N-n} \\ C_N^- &= \sum_{n=\frac{1}{2}}^{\infty} d_n^\dagger a_{N+n} - \sum_{n=\frac{1}{2}}^{\infty} a_n^\dagger b_{N+n} - \sum_{n=\frac{1}{2}}^{N-\frac{1}{2}} a_n b_{N-n} . \end{aligned} \quad (3.4)$$

The negative frequency modes may be obtained by hermitian conjugation: $C_{-N}^3 = (C_N^3)^\dagger$, $C_{-N}^+ = (C_N^-)^\dagger$ and $C_{-N}^- = (C_N^+)^\dagger$.

In this Cartan basis, the Kac-Moody algebra takes the form

$$[C_N^3, C_M^3] = N\delta_{N,-M} \quad (3.5)$$

$$[C_N^\pm, C_M^\pm] = 0 \quad (3.6)$$

$$[C_N^3, C_M^\pm] = \pm C_{N+M}^\pm \quad (3.7)$$

$$[C_N^+, C_M^-] = C_{N+M}^3 + N\delta_{N,-M} . \quad (3.8)$$

Also notice that for $N = M = 0$ the above algebra is the $SU(2)$ algebra of the charges. The algebra satisfied by the D_N^a is of course identical.

IV. GAUGE FIXING

It is most convenient in light-front field theory to choose the light-cone gauge $A^+ \equiv V = 0$. This is not possible with the boundary conditions we have imposed, however it is permissible to take $\partial_- V = 0$. In addition, we can make a further global (i.e., x^- -independent) rotation so that the zero mode of V has only a color 3 component [8,9],

$$V = v(x^+) \tau^3 . \quad (4.1)$$

At this stage the only remaining gauge freedom involves certain “large” gauge transformations, which we shall denote T_N^R and T_N^L with N any integer:

$$\begin{aligned}
T_N^R &= \exp \left[-\frac{iN\pi}{2L} x^- \tau_3 \right] \\
T_N^L &= \exp \left[\frac{iN\pi}{2L} x^+ \tau_3 \right] .
\end{aligned} \tag{4.2}$$

The combination $T_N^R T_N^L \equiv T_N$ is a gauge freedom of the theory and an example of the Gribov ambiguity [10]. We can use T_N to bring Z_R to a FMD, $-1 < Z_R < 0$. Once this is done all gauge freedom has been exhausted and the gauge fixing is completed.

The physical degrees of freedom that remain are the Fermi fields Ψ_R and Ψ_L and the gauge field zero mode Z_R , restricted to the finite interval $-1 < Z_R < 0$. All other nonvanishing components of the gauge field will be found to be constrained, as is usual in light-front field theory. Because of the finite domain of Z_R , it is convenient to use a Schrödinger representation for this variable. Thus the states of the theory will be written in the form

$$|\Phi\rangle = \zeta(Z_R) |\text{Fock}\rangle , \tag{4.3}$$

where $\zeta(Z_R)$ is a Schrödinger wavefunction and $|\text{Fock}\rangle$ is a Fock state in the fermionic variables. There remains the question of what boundary conditions should be satisfied by the wavefunction ζ . A careful analysis of the integration measure forces the wavefunction to vanish at the boundaries of the fundamental domain $\zeta(-1) = \zeta(0) = 0$.

After gauge fixing T_N is no longer a symmetry of the theory, but there is an important symmetry of the gauge-fixed theory that is conveniently studied by combining T_1 with the so-called Weyl transformation, denoted by R . Under R , $RZ_R R^{-1} = -Z_R$. R is also not a symmetry of the gauge-fixed theory, as it takes Z_R out of the fundamental domain. However, the combination $T_1 R$ is a symmetry. We find

$$T_1 R Z_R R^{-1} T_1^{-1} = -Z_R - 1 , \tag{4.4}$$

so that $T_1 R$ maps the FMD $-1 < Z_R < 0$ onto itself. In fact, it represents a reflection of the FMD about its midpoint $Z_R = -1/2$.

The action of $T_1^{R/L}$ and R on the fermion Fock operators gives rise to a spectral flow for the right handed fields,

$$\begin{aligned}
T_1^R b_n T_1^{-1R} &= b_{n-1} \quad , \quad n > 1/2 \\
T_1^R d_n T_1^{-1R} &= d_{n+1} \\
T_1^R b_{1/2} T_1^{-1R} &= d_{1/2}^\dagger \\
R b_n R^{-1} &= -d_n ,
\end{aligned} \tag{4.5}$$

and similarly for the left handed operator. The a_n and α_n are invariant under both $T_1^{R/L}$ and R . From the behavior of the Fock operator it is straight forward to deduce the behavior of the elements of the Kac Moody algebra under T^R , T^L and R and show that the algebra is invariant. We shall elaborate on the detailed implications of the symmetry $T_1 R$ when we consider the structure of the vacuum state below.

Finally, let us discuss Gauss' law and determine the rest of the vector potential in terms of the dynamical degrees of freedom. Gauss's law in matrix form is in the Cartan basis, this becomes

$$-\partial_-^2 A_3 = gJ_3^R \quad (4.6)$$

$$-(\partial_- \pm igv)^2 A_\pm = gJ_\pm^R. \quad (4.7)$$

Note that because of the gauge choice J^R only acquires a gauge correction to its 3 color component. All color components of J^L receive a gauge correction.

Eqn. (4.6) can be used to obtain the normal mode part of A_3 on the surface $x^+ = 0$:

$$A_3 = \frac{gL}{2\pi^2} \sum_{N \neq 0} \frac{C_N^3}{N^2} e^{-iN\pi x^-/L}. \quad (4.8)$$

The zero mode of Eq.(4.6) requires special attention and is discussed elsewhere [6].

Because of the restriction of Z_R to a finite domain and the boundary condition on $\zeta(Z_R)$, the covariant derivatives appearing in Eqn. (4.7) have no zero eigenvalues. Thus they may be inverted to solve for A_+ and A_- on $x^+ = 0$:

$$A_\pm = \frac{gL}{2\pi^2} \sum_N \frac{C_N^\mp}{(N \mp Z_R)^2} e^{-iN\pi x^-/L}. \quad (4.9)$$

V. VACUUM STATES OF THE THEORY

The Fock state containing no particles will be called $|V_0\rangle$. It is one of a set of states that are related to one another by T_1 transformations, and which will be denoted $|V_M\rangle$, where M is any integer. These are defined by

$$|V_M\rangle \equiv (T_1)^M |V_0\rangle, \quad (5.1)$$

where $(T_1)^{-1} = T_{-1}$. It is straightforward to determine the particle content of the $|V_M\rangle$. One finds that

$$|V_1\rangle = a_{1/2}^\dagger \beta_{1/2}^\dagger |0\rangle \quad (5.2)$$

We will focus here on the Fock states $|V_0\rangle$ and $|V_1\rangle$ since we will find that a degenerate vacuum will be constructed from them. A general discussion of $|V_M\rangle$ follows similar lines.

The combination $T_1 R$ interchanges the Fock states $|V_0\rangle$ and $|V_1\rangle$ up to a phase,

$$\begin{aligned} T_1 R |V_0\rangle &= (phase) |V_1\rangle \\ T_1 R |V_1\rangle &= (phase) |V_0\rangle. \end{aligned} \quad (5.3)$$

All of our states, as we noted previously, will be constructed from a Schrödinger wave function $\zeta(Z_R)$ and a Fock state. We construct, in the next section, the Schrödinger equation by projecting out the empty Fock state sector of the Hamiltonian. The energy eigenvalues of the system are proportional to $2L$, as one would expect, since they correspond to fluctuation of the flux around the entire spatial volume. We will make a large L approximation and only retain the ground state wave function $\zeta(Z_R)$. We chose the arbitrary phase such that,

$$T_1 R \zeta(Z_R) |V_0\rangle = e^{i\theta} \zeta(-Z_R - 1) |V_1\rangle \quad (5.4)$$

We can now construct the “ Θ vacuum ” for this theory that is invariant under the TR symmetry,

$$|\Omega\rangle = \zeta(Z_R) |V_0\rangle + e^{i\theta} \zeta(-Z_R - 1) |V_1\rangle \quad (5.5)$$

The state we have construct in Eqn. (5.5) is an eigenstate:

$$T_1 R |\Omega\rangle = |\Omega\rangle. \quad (5.6)$$

It is typically necessary to build the theory on such a vacuum state in order to satisfy the requirements of cluster decomposition as well.

VI. ENERGY-MOMENTUM TENSOR

The Poincaré generators P^- and P^+ have contributions from both the left handed and right handed fermions with Ψ_R is initialized on $x^+ = 0$ and propagates in x^+ while Ψ_L is initialized on $x^- = 0$ and propagates in x^- . These operators must therefore have contributions from both parts of the initial-value surface. Thus

$$P^\pm = \int_{-L}^L dx^- \Theta^{+\pm} + \int_{-L}^L dx^+ \Theta^{-\pm}, \quad (6.1)$$

where $\Theta^{\mu\nu}$ is the energy momentum tensor.

The right handed contribution to P^- takes its most elegant form by inserting the J^R s in term of the C^N s. We find

$$P_{rh}^- = \frac{g^2 L}{4\pi^2} \left[\sum_{N \neq 0} \frac{C_N^3 C_{-N}^3}{N^2} + \sum_N \left[\frac{C_N^+ C_{-N}^-}{(Z_R + N)^2} + \frac{C_N^- C_{-N}^+}{(Z_R - N)^2} \right] + \Pi_R^2 \right] \quad (6.2)$$

In the above equation $\Pi_R = (2\pi/g)\partial_+ v$ is the momentum conjugate to Z_R , so that $[Z_R, \Pi_R] = i$.

The object is now to find the lowest-lying eigenstates of the Hamiltonian P^- . The gauge anomaly contribution to P^- can be written as $-\frac{\pi}{2L} Q_R^2$. Acting on $|V_0\rangle$ P_{lh}^- gives zero and acting on $|V_1\rangle$ it gives $-\pi/2L$ which exactly cancels the contribution from the one left handed particle in $|V_1\rangle$. We will therefore omit the term $-\frac{\pi}{2L} Q_R^2$ and P_{lh}^- in the following discussion. It is convenient to separate P^- into a “free” part and an interaction,

$$P^- = \frac{g^2 L}{4\pi^2} [\Pi_R^2 + P_0^- + P_I^-]. \quad (6.3)$$

P_0^- includes all Z_R -dependent c -numbers and one-body Fock operators that arise from normal ordering Eq. (6.3), and has the form

$$P_0^- = C(Z_R) + V(Z_R). \quad (6.4)$$

we find,

$$C(Z_R) = -Z_R \psi'(1 + Z_R) - \psi(1 + Z_R) + Z_R \psi'(1 - Z_R) - \psi(1 - Z_R) - 2\gamma \quad (6.5)$$

where ψ is the derivative of the gamma function and γ is the Euler constant. The one body operator takes the form

$$V(Z_R) = \sum_n (A_n(Z_R) a_n^\dagger a_n + B_n(Z_R) b_n^\dagger b_n + D_n(Z_R) d_n^\dagger d_n) . \quad (6.6)$$

where

$$B_n(Z_R) = \psi'(Z_R - n + \frac{1}{2}) - \psi'(-Z_R + n + \frac{1}{2}), \quad (6.7)$$

$D_n(Z_R) = B_n(-Z_R)$ and $A_n(Z_R) = B_n(Z_R) + D_n(Z_R)$. P_I^- is a normal-ordered two body interaction. We do not display it here as it is unnecessary for our present purposes. Note also that P_0^- itself is invariant under T_1 and R . This is not true of $C(Z_R)$ and $V(Z_R)$ individually.

Consider the matrix element of P^- acting a possible vacuum state $\zeta(Z_R)|V_0\rangle$ and an arbitrary Fock state. The only non-vanishing matrix element is

$$\langle V_0 | P^- \zeta(Z_R) | V_0 \rangle = \epsilon_0 \zeta(Z_R) \quad (6.8)$$

which leads to Schrödinger equation for $\zeta(Z_R)$:

$$\left[-\frac{d^2}{dZ_R^2} + C(Z_R) \right] \zeta(Z_R) = \epsilon_0 \zeta(Z_R) . \quad (6.9)$$

The “potential” $C(Z_R)$ has a minimum at $Z_R = 0$ and diverges at $Z_R = \pm 1$. The boundary conditions are $\zeta(0) = 0$ and $\zeta(-1) = 0$. These boundary conditions are the result of a number of studies [3,8] of the behavior of states at the boundaries of Gribov regions. It is straightforward to solve this quantum mechanics problem numerically. We find that a very good fit to the numerical solution is

$$\zeta(Z_R) = -5.66 (1 - Z_R^2)^{1.63} Z_R e^{-0.835 Z_R^2} . \quad (6.10)$$

Now let us consider the state $\zeta(-Z_R - 1)|V_1\rangle$. Projecting the matrix element of $P^- \zeta(-Z_R - 1)|V_1\rangle$ with $|V_1\rangle$ we find

$$\left[-\frac{d^2}{dZ_R^2} + [C(Z_R) + D_{1/2}(Z_R)] \right] \zeta(-Z_R - 1) = \epsilon \zeta(-Z_R - 1) . \quad (6.11)$$

From the explicit forms of $C(Z_R)$ and $D_{1/2}(Z_R)$ it can be shown that $C(Z_R) + D_{1/2}(Z_R) = C(Z_R + 1)$. This is of course just the realization of the T_1 invariance of P_0^- . This explicitly demonstrates that $\Psi(-Z_R - 1)|V_1\rangle$ is a degenerate ground state of the theory.

VII. THE CONDENSATE

It is generally accepted that QCD in 1+1 dimensions coupled to adjoint Fermions develops a condensate Σ . So far Σ has only been calculated in various approximations. It has been calculated in the large- N limit [11] and in the small-volume limit for $SU(2)$ in Ref. [3]. Previously we calculated a condensate in the chiral theory with only right-handed fermions [12]. In that calculation it was the field itself that had a condensate and the result was fundamentally different from what we are considering here. It is an interesting feature of these theories that the object that condenses depends on the number of degrees of freedom in the problem. For $SU(3)$ it is a four fermion operator that has a condensate. Here we consider the vector theory with both dynamical left- and right-handed fields and we expect to obtain a condensate for $\bar{\Psi}\Psi$.

The two vacuum states $\zeta(Z_R)|V_0\rangle$ and $\zeta(-Z_R - 1)|V_1\rangle$ are both exact ground states. We have a spectral flow associated with the right-handed and left-handed operators, and thus the two physical spaces in the fundamental domain differ by one right-handed and one left-handed fermion. They effectively block diagonalize P^- sectors. One sector is built on a vacuum with no background particles and the other built on a state with background particles and these two sector only communicate through the condensate. There are no interactions in P^- that connect these blocks. Furthermore using the $T_1 R$ symmetry one can show that the blocks are identical. Therefore it is argued [4] that the condensate which is a matrix connecting these states cannot affect the massive spectrum of the theory. In ref. [5] it is pointed out that if we allow a small mass m for the adjoint fermions the theory becomes confining with a string tension σ that is related to the condensate,

$$\sigma = 2m\Sigma. \quad (7.1)$$

To calculate the condensate Σ let us consider $Tr(\bar{\Psi}(0)\Psi(0))$ and retain only the contribution of $d_{1/2}$ and $\beta_{1/2}$ particles which could give a non-zero contribution,

$$Tr(\bar{\Psi}(0)\Psi(0)) = -\frac{i}{2L\sqrt{2}} \left[d_{1/2}^\dagger \beta_{1/2}^\dagger + d_{1/2} \beta_{1/2} \right] \dots \quad (7.2)$$

Taking matrix elements with $|\Omega\rangle$ we find that only the cross terms contribute. We find for the vacuum expectation value of $\bar{\Psi}\Psi$ from Eqn. (5.5):

$$\Sigma = \langle \Omega | Tr(\bar{\Psi}(0)\Psi(0)) | \Omega \rangle = -\frac{\sin(\theta)}{2L\sqrt{2}} \int_{-1}^0 \zeta(Z_R) \zeta(-Z_R - 1) dZ_R. \quad (7.3)$$

Evaluating the integral numerically we find,

$$\Sigma = \frac{\sin(\theta)}{2L} .656 \quad (7.4)$$

We see that this expression behave like $1/L$. This is a common result for discrete light cone calculations and is found even in the Schwinger model where the exact result is known not to have this behavior. This a result of the crude treatment of the small p^+ region that occurs in this method. The region $p^+ = 0$ is generally very singular and it is believed that

in the limit $L \rightarrow \infty$ one encounters these singularities and they cancel the vanishing $1/L$ leaving a finite result. We also note that the only other calculation [3] of this quantity, done in different gauge, in a small volume approximation and discretized and quantized at equal time, finds this $1/L$ behavior.

Kutasov and Schwimmer [4] have pointed out that there are classes of theories that have the same massive spectrum. For example QCD_{1+1} coupled to two flavors of fundamental fermions should have the same massive spectrum as the theory we have considered here. However the theory with fundamental fermions will not have a condensate. The primary conditions for this universality is the decoupling of the left and right handed fields.

VIII. CONCLUSIONS

We have shown that in QCD coupled to chiral adjoint fermions in two dimensions the light-front vacuum is two-fold degenerate as one would expect on general grounds. The source of this degeneracy is quite simple. Because of the existence of Gribov copies, the one gauge degree of freedom, the zero mode of A^+ , must be restricted to a FMD. The domain of this variable, which after normalization we call Z_R , is bounded by the integers. Furthermore there is a $T_1 R$ symmetry which is effectively a reflection about the midpoint of the FMD. The $T_1 R$ symmetry operator acting on the Fock vacuum generates a second degeneracy vacuum. Since T_1 generates a spectral flow for the fermions this second vacuum states contains one left-handed and one right-handed fermion each with momentum $\frac{\pi}{2L}$, but the state can be shown to still have $P^+ = P^- = 0$.

We form the analog of a θ vacuum from these two-fold degenerate vacuum states which respects all of the symmetries of the theory. We find that $Tr(\bar{\Psi}\Psi)$ has a vacuum expectation value with respect to this θ vacuum and we find an exact expression for this condensate Σ . It is unlikely that the condensate we find here is equivalent to the theories that have been studied in the equal-time formulation [11,3], because the infrared regulator used there appears to couple the left and right-handed fermions.

Since P^- is block diagonal in our degenerate vacuum states the condensate does not affect the massive spectrum of the theory and the theory has no massless bound state. However this condensate is proportional to the string tension [5] where the adjoint Fermions are given a small mass.

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REFERENCES

- [1] E. Witten, Nuovo Cim. A **51**, 325 (1979).
- [2] A. Smilga, Phys. Rev. D **49**, 6836 (1994).
- [3] F. Lenz, M. Shifman and M. Thies, Phys. Rev. D **51**, 7060 (1995).
- [4] D. Kutasov and A. Schwimmer, Nucl. Phys. B **442**, 447 (1995).
- [5] D. J. Gross, I. R. Klebanov, A. V. Matytsin and A. Smilga, [hep-th/9511104](#).
- [6] S. Pinsky D. Robertson and G. McCartor, "SU(2) Yang-Mills Theory in 1+1 Dimensions Coupled to Massless Adjoint Fermions on The Light-Cone" Ohio-State Preprint in preparation.
- [7] P. Goddard and D. Olive, Int. J. Mod. Phys. A **2**, 303 (1986).
- [8] A. C. Kalloniatis, H.-C. Pauli and S. S. Pinsky, Phys. Rev. D **50**, 6633 (1994).
- [9] S. S. Pinsky and A. Kalloniatis, Phys. Lett. **B365** 225 (1995).
- [10] V.N. Gribov, Nucl. Phys. **B** 139 (1978) 1.
- [11] I. I. Kogan and A. R. Zhitnitsky, OUTP-95-25 P, [hep-ph/9509322](#); A. R. Zhitnitsky, Phys. Lett. B **165**, 405 (1985); Sov. J. Phys. **43**, 999 (1986); *ibid.* **44**, 139 (1986).
- [12] S.S. Pinsky and D. G. Robertson "Light-Front QCD_{1+1} Coupled to Chiral Adjoint Fermions, Phys. Lett. B379 (1996) 169